

DARK ENERGY, DARK MATTER AND THE CHAPLYGIN GAS

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Abstract

The possibility that the dark energy may be described by the Chaplygin gas is discussed. Some observational constraints are established. These observational constraints indicate that a unified model for dark energy and dark matter through the employment of the Chaplygin gas is favored.

PACS number(s): 98.80.Bp, 98.65.Dx

The possible existence of dark matter and dark energy in the Universe constitutes one of the most important problems in physics today. One calls dark matter the mysterious component of galaxies and clusters of galaxies that manifests itself only through gravitational effects, affecting only the dynamics of those object. Dark matter appears in collapsed objects. The existence of dark energy, on the other hand, has been speculated due to the observation of the Universe as a whole. In particular, the position of the first accoustic peak in the spectrum of the anisotropy of the cosmic microwave background radiation[1] and the data from supernova type Ia[2, 3] suggest that the dynamics of the Universe is such that the most important matter component is non-clustered, exhibiting negative pressure. The current observations favor $\Omega_{dm} \sim 0.3$ and $\Omega_{de} \sim 0.7$ for the proportion of dark matter and dark energy.

The most natural candidate to represent dark energy is a cosmological constant[4]. However, it is necessary a fine tuning of 120 orders of magnitude in order to obtain agreement with observations. Another popular candidate today is quintessence, a self-interacting scalar field[5, 6, 7]. But the quintessence program suffers from fine tuning of microphysical parameters. In this work, we discuss the possibility that the dark energy may be represented by the Chaplygin gas[8, 9, 10, 11, 12], which is characterized by the equation of state

$$p = -\frac{A}{\rho} \quad . \quad (1)$$

The Chaplygin gas has an interesting motivation connected with string theory. In fact, if we consider a d -brane configuration in the $d+2$ Nambu-Goto action, the employment of the light-cone parametrization leads to the action of a newtonian fluid with the equation of state (1), whose symmetries are the same as the Poincaré group. Hence, the relativistic

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character of the action is somehow hidden in the equation of state (1). For a review of the properties of the Chaplygin gas see reference [13].

Using the relativistic equation of conservation for a fluid in a FRW background, we obtain that the density of the Chaplygin gas depends on the scale factor as

$$\rho = \sqrt{A + \frac{B}{a^6}} \quad . \quad (2)$$

Hence, initially the Chaplygin gas behaves as a pressureless fluid and assumes asymptotically a behaviour typical of a cosmological constant. In this sense, the Chaplygin gas may be an alternative for the description of dark energy. Remark that, even if the pressure associated with the Chaplygin gas is negative, the sound velocity is real. Hence, no instability problem occurs as it happens with other fluids with negative pressure[14].

In reference [9] the density perturbations in a Universe dominated by the Chaplygin gas was investigate. In that work, the fact that the Chaplygin gas may be represented by a newtonian fluid was explored. The density contrast behaves as

$$\delta = t^{-1/6} \left[C_1 J_\nu(\Sigma^2 n t^{7/3}) + C_2 J_{-\nu}(\Sigma^2 n t^{7/3}) \right] \quad , \quad (3)$$

where n is the wavenumber of the perturbations, $\nu = 5/14$ and $\Sigma^2 = \frac{54}{49} \frac{A\pi G}{a_0^2}$, a_0 is a constant and C_1 and C_2 are constants that depend on n . A simple analysis of this solution indicates that initially the perturbations behave as in the case of the pressureless fluid, presenting asymptotically oscillatory behaviour with decreasing amplitude, approaching a zero value typical of a cosmological constant. Even if this result was obtained in a newtonian context, a numerical analysis of the corresponding relativistic equations exhibit the same behaviour.

In reference [10], the mass power spectrum of the perturbations in a Universe dominated by the Chaplygin gas was computed. The matter content was assumed to be composed of radiation, dark matter and dark energy, the latter represented by the Chaplygin gas. Due to the complexity of perturbed equations, a numerical analysis was performed. The initial spectrum, at the moment of decoupling of radiation and matter, was supposed to be the Harrison-Zeldovich scale invariant spectrum. Taken at constant time, this implies to impose an initial spectral index $n_s = 5$. The system evolves and the power spectrum is computed for the present time. The final results were obtained for a Universe dominated uniquely by a pressureless fluid (*baryonic model*), for a cosmological constant model and for a mixed of dark matter and Chaplygin gas, with different values for the sound velocity of the Chaplygin gas, defined as $v_s^2 = \bar{A} = A/\rho_{c0}^2$, where ρ_{c0} is the Chaplygin gas density today. When $\bar{A} = 1$ the Chaplygin gas becomes identical to a cosmological constant. It was verified that the Chaplygin gas models interpolate the baryonic model and the cosmological constant model as \bar{A} varies from zero to unity. The results for the power spectrum $P(n) = n^{3/2} \delta_n$ in terms of n/n_0 , where n_0 is reference scale corresponding to 100 Mpc , are displayed in figure 1 for the baryonic model, cosmological constant model and for the Chaplygin gas model with different values for \bar{A} . The spectral indice for perturbations of some hundreds of megaparsecs up to the Hubble radius is $n_s \sim 4.7$

for the baryonic model, $n_s \sim 4.2$ for the cosmological constant model, $n_s \sim 4.5$ for the Chaplygin gas with $\bar{A} = 0.5$.

Another important test to verify if the Chaplygin gas model may represent dark energy is the comparison with the supernova type Ia data. In order to do so, we evaluate the luminosity distance[15] in the Chaplygin gas cosmological model. In such a model, the luminosity distance, for a flat Universe, reads

$$D_L = (1+z) \int_0^z \frac{dz'}{\sqrt{\Omega_{m0}(1+z')^3 + \Omega_{c0}\sqrt{\bar{A} + (1-\bar{A})(1+z')^6}}} \quad , \quad (4)$$

z being the redshift. From this expression the following relation between the apparent magnitude m and the absolute magnitude M is obtained:

$$m - M = 5 \log \left\{ (1+z) \int_0^z \frac{dz'}{\sqrt{\Omega_{m0}(1+z')^3 + \Omega_{c0}\sqrt{\bar{A} + (1-\bar{A})(1+z')^6}}} \right\} \quad . \quad (5)$$

In order to compare with the supernova data, we compute the quantity

$$\mu_0 = 5 \log \left(\frac{D_L}{Mpc} \right) + 25 \quad . \quad (6)$$

The quality of the fitting is characterized by the parameter

$$\chi^2 = \sum_i \frac{[\mu_{0,i}^o - \mu_{0,i}^t]^2}{\sigma_{\mu_0,i}^2 + \sigma_{mz,i}^2} \quad . \quad (7)$$

In this expression, $\mu_{0,i}^o$ is the measured value, $\mu_{0,i}^t$ is the value calculated through the model described above, $\sigma_{\mu_0,i}^2$ is the measurement error, $\sigma_{mz,i}^2$ is the dispersion in the distance modulus due to the dispersion in galaxy redshift due to peculiar velocities. This quantity we will taken as

$$\sigma_{mz} = \frac{\partial \log D_L}{\partial z} \sigma_z \quad , \quad (8)$$

where, following [3], $\sigma_z = 200 \text{ km/s}$. We evaluate, in fact, χ_ν^2 , the estimated errors for degree of freedom.

The best fitting was obtained for the case where $\Omega_{m0} = 0$, $\Omega_{c0} = 1$, $\bar{A} \sim 0.847$, $H_0 \sim 62.1$, with $T_0 \sim 15.1 \text{ Gy}$ for the age of Universe. In this case, $\chi_\nu^2 \sim 0.760$. As a comparison, if we substitute the Chaplygin gas by a cosmological constant ($\bar{A} = 1$), the best fitting is given by $\Omega_{m0} \sim 0.244$, $\Omega_\Lambda \sim 0.756$, $H_0 \sim 61.8$, and the age of the Universe is $T_0 \sim 16.5 \text{ Gy}$. In this model $\chi_\nu^2 \sim 0.774$. In figure 2 we display this fitting.

These results deserve some comments. First, if we consider a cosmological constant model, the best fitting is in quite good agreement with other observational data, mainly those from *CMB* and the dynamics of clusters of galaxies. However, the Chaplygin gas model indicates no need of dark matter, and the fitting quality is slightly better than the cosmological model. This is interesting since there are claims that the Chaplygin gas

may unify dark matter with dark energy: at small scales, the Chaplygin gas may cluster, playing the role of dark matter, remaining a smooth component at large scale [8, 11, 12].

The results presented above indicate that the Chaplygin gas may be taken seriously as a candidate to describe dark energy and dark matter. However, more analysis are needed in order to verify to which extent it can be a better alternative with respect to the cosmological constant model combined with dark energy. A crucial test is the analysis of the spectrum of anisotropy of the cosmic microwave background radiation.

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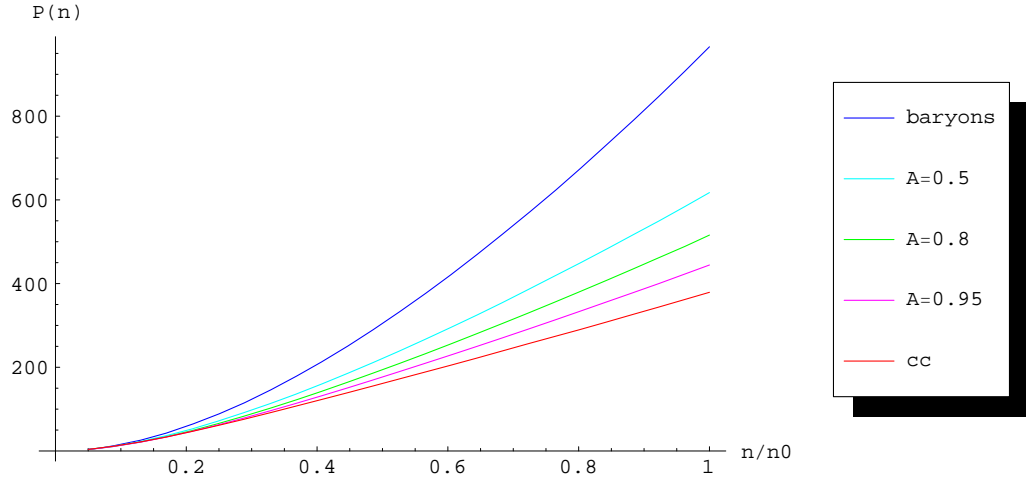


Figure 1: The power spectrum $P(n) = n^{3/2}\delta_n$ as function of n/n_0 .

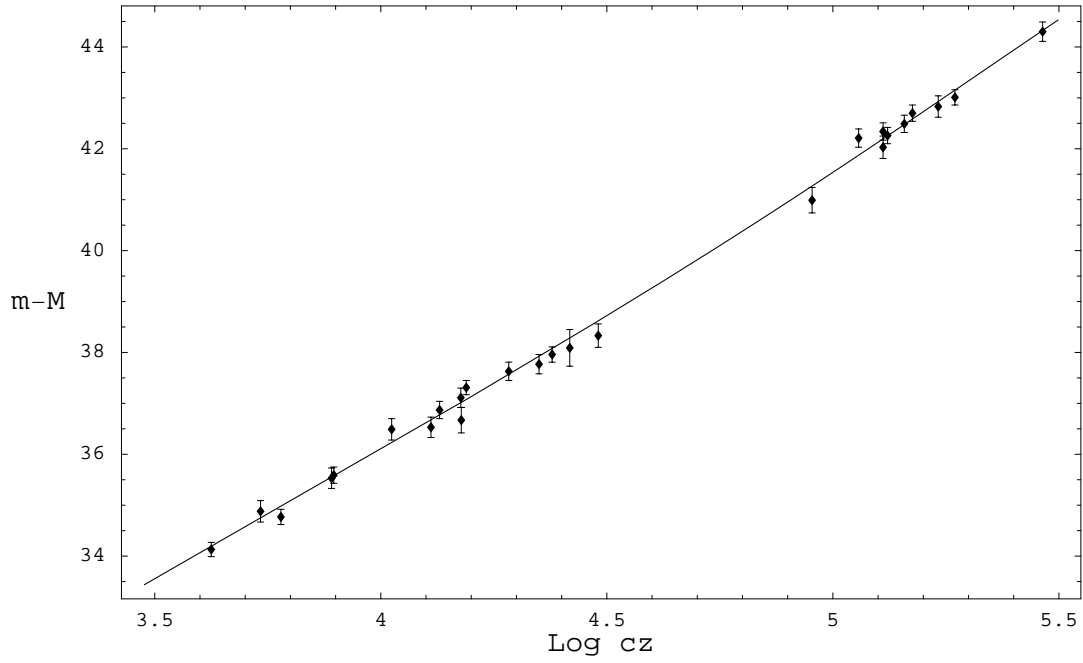


Figure 2: The the best fitting model, where $\Omega_{m0} = 0$, $\Omega_{c0} = 1$, $\bar{A} \sim 0.847$ and $H_0 \sim 62.1$.